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Application of Laplace Adomian Padé approximant to solve exponential stretching sheet problem in fluid mechanics

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ABSTRACT: The purpose of this study is to apply Laplace Adomian Decomposition Method (LADM) for obtaining the analytical and numerical solutions of a nonlinear differential equation that describes a magnetohydrodynamic (MHD) flow under the stretching sheet problem. By using this method, the similarity solutions of the problem are obtained. For obtaining computational solutions, we combined the obtained series solutions by the LADM with the Padé approximation. The method is easy to apply and give high accuracy results. From the tables and figures efficiency of the presented technique is shown.

Keywords: Laplace transformation; Adomian Decomposition Method; Pade approximation;Navier–Stokes equations; Semi-infinite interval; MHD flow.

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INTRODUCTION

Nonlinear phenomena, that appear in many ares in scientific fields such as solid state physics, plasma physics, fluid mechanics, population models and chemical kinetics, can be defined by nonlinear differential equations. One of the most important kind of these equations is the nonlinear differential equations that characterize boundary layers in unbounded domain.

Firstly, Sakiadis in 1962 (Sakiadis, 1961) solved the problem of forced convection along an isothermal constantly moving plate which it is a classical problem of fluid mechanics. Magnetohydrodynamic (MHD) is considering the interaction conducting fluids with electromagnetic problms. The flow on electrically conducting fluid with in the magnetic field is one of the most applicable sections in various areas of engineering and technology. The viscous flow du to the stretching boundary is important in extrusion process when sheet material is pulled out of an orifice with rasing velocity. Therefore, since the numerical/analytical of fluid flow across a thin liquid filmis very important in many branches of science and technology, then many authors paid much attention to consider the behavior of this problem numerically and analytically. In investigation of boundary layers problems, by applying a good variable transformation, we convert the system of Navier-Stokes equations to a nonlinear ordinary boundary value problem with a semi-infinite interval. In (Boyd, 2000) , the infinite domain replaced by $[-L, L]$ and the semi-infinite travel with $[0,L]$ by selecting a sufficiently large L . Guo (Guo, 2000) converted the problem of semi-infinite domain to a model of boundary domain.

Recently, theory of the boundary layers has been successfully used and investigated to the MHD Falkner-Skan flow of viscous fluids (Falkner, 1931; Soundalgekar, 1981). Very recently, Robert et.al analyzed the existence and uniqueness results of the MHD Falkner-Skan flow (Robert et al., 2010) Abbasbandy et.al in (Abbasbandy et al., 2009; Abbasbandy et al., 2009) investigated this problem numerically by using Hankel-Padé method and Homotopy analysis method, respectively. Afzal in (Noor, 2010) studied the Falkner–Skan problem for flow past a stretching surface with suction or blowing. Very recently, the MHD Falkner-Skan boundary layer flow over a permeable wall in the presence of a transverse magnetic field authors have been examined and approximate results for the similarity solutions have been obtained by using the differential transform method (DTM) coupled with the Padé approximation (Xiao-hong, 2011).

The boundary-layer problems often can be expressed in the form of a nonlinear two point boundary value problem with specific conditions at the two boundaries of the domain $D=[0,\infty)$, which can not be often solved analytically and exactly in a closed form.

The Adomian Decomposition Method has been applied to a wide class of problems in physics, biology and chemical reactions. The method provides the solution in a rapid convergent series with computable terms (Adomian, 1994, Adomian et al., 1996). Then by applying this method, the numerical solutions of some equations can be obtained. In this research, we will combine the Adomian Decomposition Method with the laplace transformation to obtain the similarity solution of an important nonlinear differential equation. This method is proposed in (Khuri, 2001; Syam, 2006). Then, a combination of the Laplace Adomian Decomposition Method with the Padé approximations, has been presented by Bakera (Baker, 1975).

This paper has the following structer: converting the model of system of nonlinear PDEs to the nonlinear ordinary differential equation is presented in section 2. In section 3, we applied the LADM to the obtained ordinary differential equation. In section 4, combining the LADM with the Padé approximant is shown. Finally, the numerical result is reported.

Mathematical formulation and discussion

In this section suppose that we consider flaw of an incompressible viscous fluid over exponential stretching sheet at $y=0$. Suppose that (u,υ) be the velocity components in the (x,y) directions, respectively. In fact, it is the kinematic viscosity which is the ratio of dynamic viscosity to the density of the luid i.e. $\upsilon=\frac{\mu}{\rho}$.

Then based on the above assumption, the corresponding phenomenon can be introduced as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}, \qquad (2)$$

subject to the following boundary conditions

$$\begin{cases} u=U_0e^{\frac{x}{L}},\ \upsilon=0,\ at\ y=0, \\ u\to 0,\ as\ y\to\infty, \end{cases} \qquad (3)$$

where U_0 and L are the reference velocity a constant respectively.

Now we define the following similarity transformations

$$\begin{cases} \eta=\sqrt{\frac{U_0}{2\upsilon l}}e^{x}Ly, \\ u=U_0e^{x}Lf'(\eta), \\ \upsilon=-\sqrt{\frac{\upsilon U_0}{2L}}e^{x}2L(f(\eta)+\eta f(\eta)), \end{cases} \qquad (4)$$

By using (4) in the (1), (2) and (3), we have

$$f'''(\tau)-2f'^2(\tau)+f(\tau)f''(\tau)=0, \qquad (5)$$

with boundary conditions

$$\begin{cases} f(0)=0, \\ f'(0)=1, \\ \lim\limits_{\tau\to\infty}=0. \end{cases} \qquad (6)$$

Based on our best knowledge, the numerical solution of (5) and (6) has been discussed in [32]. In this paper, we shall proposed one another numerical scheme based on Laplace Adomian decomposition method for analytical solution and Padé aproximant for numerical solution of (5) in the presence of boundary condition (6).

Application of the Laplace Adomian Decomposition Method

In this section, we use the Laplace transform algorithm for third order nonlinear initial value problem to have similarity solution of the nonlinear ordinary differential equation (5) with boundary conditions(6). Firstly, we take the Laplace transformation (L) on both sides of Eq.(5) in the presence of the boundary conditions (6). Then, we get

$$\begin{aligned}&(s^3 L\{f(\tau)\} - s^2 f(0) - sf'(0) - f''(0))\\&-2L\{f'(\tau)^2\} + L\{f(\tau)f''(\tau)\}) = 0.\end{aligned} \tag{7}$$

Within the our computational process, we will determine the value of $f''(0)$. Then, if we define $f''(0)=\alpha$, we can solve the equation (5) subject to the following initial value conditions

$$\begin{cases} f(0) = 0, \\ f'(0) = 1, \\ f''(0) = \alpha, \end{cases} \tag{8}$$

where α is an unknown constant that it must to be found. Now, by applying the conditions (8) into (7), we obtain

$$L\{f(\tau)\} = \frac{1}{s^3}(\alpha + s + 2L\{f'(\tau)^2\} - L\{f(\tau)f''(\tau)\}). \tag{9}$$

By using the Laplace decomposition method, we will be able to obtain an analytical solution of (9) in the form of the following infinite series

$$f(\tau) = \sum_{n=0}^{\infty} f_n(\tau). \tag{10}$$

By presenting an iterative process, we will find the components $f_n(\tau)$, for $n = 0,1,2,...$. In addition, we decomposed the nonlinear terms $f'(\tau)^2$ and $f(\tau)f''(\tau)$ defined into Adomian polynomials (Adomian 1994, Adomian et.al 1996) to the following cases

$$\begin{cases} (f'(\tau))^2 = \sum_{n=0}^{\infty} A_n(\tau), \\ f(\tau)f''(\tau) = \sum_{n=0}^{\infty} B_n(\tau), \end{cases} \tag{11}$$

Then, we have

$$\begin{cases} A_n(\tau) = \sum_{i=0}^{n} f_i'(\tau) f_{n-i}'(\tau), \\ B_n(\tau) = \sum_{i=0}^{n} f_i(\tau) f_{n-i}''(\tau), \end{cases} \tag{12}$$

By using above relations, the few components of the Adomian polynomials of above nonlinear terms cab be given as follow:

$$\begin{cases} A_0(\tau) = f_{0'}^2(\tau), \\ A_1(\tau) = 2 f_0'(\tau) f_1'(\tau), \\ A_2(\tau) = f_1'^2(\tau) + 2 f_0'(\tau) f_2'(\tau), \\ A_3(\tau) = 2 f_0'(\tau) f_3'(\tau) + 2 f_1'(\tau) f_2'(\tau), \\ A_4(\tau) = f_2'^2(\tau) + 2 f_0'(\tau) f_4'(\tau) + 2 f_1'(\tau) f_3'(\tau), \\ \vdots \end{cases} \tag{13}$$

and

$$\begin{cases} B_0(\tau) = f_0(\tau) f_0'(\tau), \\ B_1(\tau) = f_1(\tau) f_0'(\tau) + f_0(\tau) f_1'(\tau), \\ B_2(\tau) = f_0(\tau) f_2'(\tau) + f_2(\tau) f_0'(\tau) + f_1(\tau) f_1'(\tau), \\ B_3(\tau) = f_0(\tau) f_3'(\tau) + f_1(\tau) f_2'(\tau) + f_2(\tau) f_1'(\tau) + f_3(\tau) f_0'(\tau), \\ B_4(\tau) = f_0(\tau) f_4'(\tau) + f_1(\tau) f_3'(\tau) + f_2(\tau) f_2'(\tau) + f_3(\tau) f_1'(\tau) + f_4(\tau) f_0'(\tau), \\ \vdots \end{cases} \tag{14}$$

In this way, by using the above results and Adomian polynomials into (9), we get

$$\mathrm{L}\{\sum_{n=0}^{\infty} f_n(\tau)\} = \frac{1}{s^3}(s + \alpha + 2\mathrm{L}\{\sum_{n=0}^{\infty} A_n\} - \mathrm{L}\{\sum_{n=0}^{\infty} B_n\}). \tag{15}$$

Then, we have

$$\mathrm{L}\{\sum_{n=0}^{\infty} f_n(\tau)\} = K(s) + \frac{1}{s^3}(2\mathrm{L}\{\sum_{n=0}^{\infty} A_n\} - \mathrm{L}\{\sum_{n=0}^{\infty} B_n\}), \tag{16}$$

where $K(s) = \frac{}{s^2} + \frac{\alpha}{s^3}$ represents the term arising from prescribe initial conditions, then based on the modified Laplace decomposition method [?], we can decompose the function $K(s)$ into two parts as the $K(s) = K_0(s) + K_1(s)$. Therefore, for obtaining the $f_n, (n \geq 0)$, firstly we compare both sides of the equation (15) and then use from the inverse Laplace transform L^{-1}. Finally, by applying the following iterative process, we can obtain the values of f_n for $n = 0,1,2,...$

$$\begin{cases} f_0 = \mathrm{L}^{-1}\{\frac{1}{s^3}(s)\}, \\ f_1 = \mathrm{L}^{-1}\{\frac{1}{s^3}(\alpha + 2\mathrm{L}\{A_0\} - \mathrm{L}\{B_0\})\}, \\ f_2 = \mathrm{L}^{-1}\{\frac{1}{s^3}(2\mathrm{L}\{A_1\} - \mathrm{L}\{B_1\})\}, \\ f_3 = \mathrm{L}^{-1}\{\frac{1}{s^3}(2\mathrm{L}\{A_2\} - \mathrm{L}\{B_2\})\}, \\ f_4 = \mathrm{L}^{-1}\{\frac{1}{s^3}(2\mathrm{L}\{A_3\} - \mathrm{L}\{B_3\})\}, \\ f_5 = \mathrm{L}^{-1}\{\frac{1}{s^3}(2\mathrm{L}\{A_4\} - \mathrm{L}\{B_4\})\}, \\ f_6 = \mathrm{L}^{-1}\{\frac{1}{s^3}(2\mathrm{L}\{A_5\} - \mathrm{L}\{B_5\})\}, \\ f_7 = \mathrm{L}^{-1}\{\frac{1}{s^3}(2\mathrm{L}\{A_6\} - \mathrm{L}\{B_6\})\}, \\ f_8 = \mathrm{L}^{-1}\{\frac{1}{s^3}(2\mathrm{L}\{A_7\} - \mathrm{L}\{B_7\})\}, \\ f_9 = \mathrm{L}^{-1}\{\frac{1}{s^3}(2\mathrm{L}\{A_8\} - \mathrm{L}\{B_8\})\}, \\ f_{10} = \mathrm{L}^{-1}\{\frac{1}{s^3}(2\mathrm{L}\{A_9\} - \mathrm{L}\{B_9\})\}, \\ f_i = \mathrm{L}^{-1}\{\frac{1}{s^3}(2\mathrm{L}\{A_{i-1}\} - \mathrm{L}\{B_{i-1}\})\}, \ i = 10, 11, ..., \end{cases} \tag{17}$$

By using inverse Laplace transform in the equation (17), we can obtain the initial term f_0. Now, we can compute the value of f_1 by using the known value of f_0. By continuing this process, we can find the successive terms. Then, we have

$$
\begin{cases}
f_0(\tau) = \tau, \\
f_1(\tau) = \frac{1}{2}\alpha\tau^2 + \frac{1}{3}\tau^3, \\
f_2(\tau) = \frac{1}{8}\alpha\tau^4 + \frac{1}{30}\tau^5, \\
f_3(\tau) = \frac{1}{40}\alpha^2\tau^5 + \frac{19}{720}\alpha\tau^6 + \frac{2}{315}\tau^7, \\
f_4(\tau) = \frac{3}{560}\alpha^2\tau^7 + \frac{167}{40320}\alpha\tau^8 + \frac{13}{22680}\tau^9, \\
f_5(\tau) = \frac{3}{4480}\alpha^3\tau^8 + \frac{527}{362880}\alpha^2\tau^9 + \frac{17}{26880}\alpha\tau^{10} + \frac{2}{22275}\tau^{11}, \\
f_6(\tau) = \frac{31}{134400}\alpha^3\tau^{10} + \frac{10183}{39916800}\alpha^2\tau^{11} + \frac{44399}{479001600}\alpha\tau^{12} + \frac{37}{3891888}\tau^{13} \\
f_7(\tau) = \frac{31}{1478400}\alpha^4\tau^{11} + \frac{14447}{239500800}\alpha^3\tau^{12} + \frac{292163}{6227020800}\alpha^2\tau^{13} + \frac{1124731}{87178291200}\alpha\tau^{14} \\
\quad + \frac{802}{638512875}\tau^{15}, \\
f_8(\tau) = \frac{569}{76876800}\alpha^4\tau^{13} + \frac{573871}{43589145600}\alpha^3\tau^{14} + \frac{9686063}{1307674368000}\alpha^2\tau^{15} \\
\quad + \frac{485237}{278970531480}\alpha\tau^{16} + \frac{192121}{1389404010600}\tau^{17}, \\
f_9(\tau) = \frac{569}{1076275200}\alpha^5\tau^{14} + \frac{111941}{46702656000}\alpha^4\tau^{15} + \frac{6041801}{2324754430200}\alpha^3\tau^{16} \\
\quad + \frac{59632897}{50812489782000}\alpha^2\tau^{17} + \frac{1487423263}{6402373707528000}\alpha\tau^{18} + \frac{2501}{142781302215}\tau^{19}, \\
f_{10}(\tau) = \frac{657}{2609152000}\alpha^5\tau^{16} + \frac{7892897}{13680285669000}\alpha^4\tau^{17} + \frac{274174541}{582033973428000}\alpha^3\tau^{18} \\
\quad + \frac{4242353851}{24329020018766400}\alpha^2\tau^{19} + \frac{624854521}{20444554609560000}\alpha\tau^{20} + \frac{99375481}{49893498241560000}\tau^{21}, \\
\vdots
\end{cases}
$$

By obtaining the components $f_i(\tau)$, for $i = 0,1,2,3,...$, the approximate analytic solution of the unknown function $f(\tau)$ can be found from equation (10). Then, the approximate analytic solution for the ten iteration step is

$$
\begin{aligned}
f(\tau) = \sum_{n=0}^{10} f_n(\tau) = \\
\tau + \frac{1}{2}\alpha\tau^2 + \frac{1}{3}\tau^3 + \frac{1}{8}\alpha\tau^4 + \frac{1}{30} + \frac{1}{40}\alpha^2)\tau^5 + \frac{19}{720}\alpha\tau^6 + \frac{2}{315} + \frac{3}{560}\alpha^2)\tau^7 \\
+ (\frac{167}{40320}\alpha + \frac{3}{4480}\alpha^3\tau^8 + \frac{13}{22680} + \frac{527}{362880}\alpha^2\tau^9 + (\frac{17}{26880}\alpha + \frac{31}{134400}\alpha^3)\tau^{10} \\
+ (\frac{2}{22275} + \frac{10183}{39916800}\alpha^2 + \frac{31}{1478400}\alpha^4)\tau^{11} + (\frac{44399}{479001600}\alpha + \frac{14447}{239500800}\alpha^3\tau^{12} \\
+ \frac{37}{3891888} + \frac{569}{76876800}\alpha^4 + \frac{292163}{6227020800}\alpha^2)\tau^{13} \\
+ (\frac{1124731}{87178291200}\alpha + \frac{573871}{43589145600}\alpha^3 + \frac{569}{1076275200}\alpha^5)\tau^{14} \\
+ \frac{802}{638512875} + \frac{9686063}{1307674368000}\alpha^2 + \frac{111941}{46702656000}\alpha^4)\tau^{15} \\
+ (\frac{485237}{278970531480}\alpha + \frac{6041801}{2324754430200}\alpha^3 + \frac{657}{2609152000}\alpha^5)\tau^{16} \\
+ (\frac{59632897}{50812489782000}\alpha^2 + \frac{192121}{1389404010600} + \frac{7892897}{13680285669000}\alpha^4)\tau^{17} \\
+ (\frac{274174541}{582033973428000}\alpha^3 + \frac{1487423263}{6402373707528000}\alpha)\tau^{18} \\
+ (\frac{2501}{142781302215} + \frac{4242353851}{24329020018766400}\alpha^2)\tau^{19} \\
+ \frac{624854521}{20444554609560000}\alpha\tau^{20} + \frac{99375481}{49893498241560000}\tau^{21}
\end{aligned}
\qquad (18)
$$

From Eq (18), it is evident that the obtained analytic solutions through LADM are power series in the independent variable. Then, according to boundary condition $f'(\infty)=0$, these solutions have not the correct behavior at infinity and so these solutions can not be directly applied. So, to resolve this problem, we combine the series solutions, obtained by the LADM, with the Padé approximants.

The LADM-Padé Approximation

Combining the obtained series solutions by the LADM in the previous section with the Padé approximation is the main part of this section. To this end, we apply this process for obtaining some high accuracy computational results for problem (5) with boundary conditions (6). Then, we transform the power series obtained by the Laplace Adomian Decomposition Method (18) into a rational function as follow

$$[S/N](\tau)=\frac{\sum_{j=0}^{S}a_j\tau^j}{1+\sum_{j=1}^{N}b_j\tau^j}. \qquad (19)$$

We know that if $N\geq S$, then the limit at infinity in the boundary conditions (6) has a correct behavior. So the rational function (19) has $S+N+1$ coefficient that we can select them. If $[S/N](\tau)$ is exactly a Padé approximation, then $f(\tau)-[S/N](\tau)=O(\tau^{S+N+1})$. Then we can obtain the coefficient a_j and b_j by the following relations

$$\sum_{i=0}^{j}b_i f_{j-i}=a_j, \quad j=0,\ldots,S, \qquad (20)$$

$$\sum_{i=0}^{j}b_i f_{j-i}=0, \quad j=S+1,\ldots,S+N, \qquad (21)$$

where $a_k-b_k=0$ if $k>N$. From (20) and (21), we can obtain the values of $a_i(0\leq i\leq S)$ and $b_j(1\leq j\leq N)$.

We know that if the function $f(t)$ is bounded i.e. for all $t>0$ we have $|f(t)<M$ and the $\lim_{t\to\infty}f(t)=f(\infty)$ be exist, then $\lim_{s\to\infty}F(s)=f(\infty)$, where $F(s)=L(f(t))$, the laplace transform of the function $f(t)$.

For finding the unknown parameter α, we can utilize the above point for laplace transform to $f'(t)$ or by using Pade sequences.

Plot of the approximate solution of $f(\tau)$ and $f'(\tau)$, which obtained by the LADM-Pade is shown in Figure 1 and Figure 2. The accuracy of proposed method can be understand from these plots.

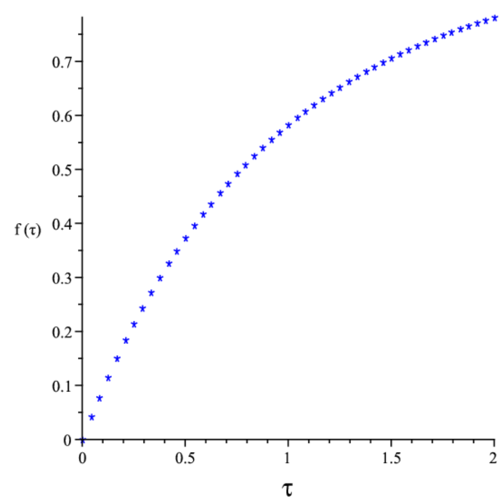

Figure 1. Plot of LADM-Padé approximate solution of $f(\tau)$

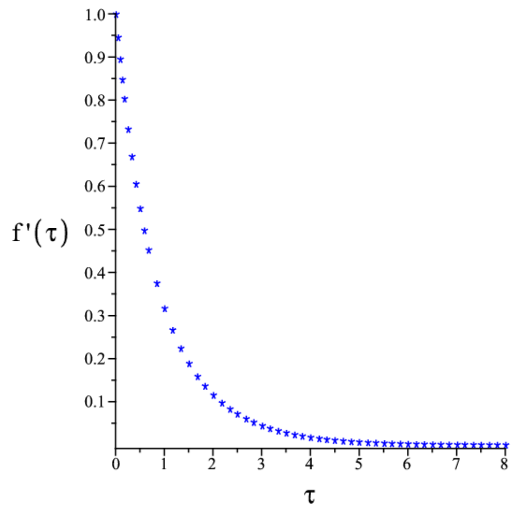

Figure 2.

CONCULSION

In this article, one of the third order nonlinear autonomous equation subject to a boundary condition which defined at infinity, is considered. A new semi-analytical method (Laplace Adomian Decomposition coupled with Pade approximant) is successfully aplied for solving this equation. The obtained computational results by using our method are presented in a table. It is evidence that this method give high accuracy results in very few itterations and can be applied to other similar problems.

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